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Time to Recruitment with Correlated Loss of Manpower under Different Renewal Process for Exit and Breaking Decisions Forms a Modified Renewal Process

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Abstract-A stochastic model is considered in this paper to determine the mean and variance of time to recruitment for a single grade manpower system. It is assumed that the depletion of manpower in the organization is classified into voluntary, involuntary exit of personnel and frequent breaks taken by the existing workers in the organization. The loss of manpower due to voluntary exits are assumed to form a sequence of exchangeable and constantly correlated exponential random variables. By assuming that inter-voluntary exit times and inter-involuntary exit times forms different renewal process and the inter-breaking decision times forms modified renewal process, variance of time to recruitment is determined for three different cases of inter-voluntary exit times using a univariate CUM policy of recruitment. The results are numerically illustrated by assuming specific distributions. The influence of the nodal parameters on the mean and variance of time to recruitment are studied and relevant conclusions are presented.

Index Terms-Single grade Manpower system, Correlated loss of manpower, Inter-voluntary exit times, Inter-involuntary exit times, Inter-breaking decision times, Different renewal process, Modified Renewal process, Univariate CUM Policy.

1. INTRODUCTION

Resignation, retirement and death are the common phenomenon of loss of manpower in the organization. The exodus is possible whenever the organization announces revised policies regarding sales target, revision of wages, incentives perquisites etc. This leads to reduction in the total strength of marketing personnel and will adversely affect the sales turnover of the organization, if recruitment is not planned. Frequent recruitment may also be expensive due to the cost of recruitment and training. As the loss of manpower is unpredictable, suitable recruitment policy has to be designed to overcome this loss. A univariate recruitment policy, usually known as CUM policy of recruitment [3] and [7] are based on the replacement policy associated with the shock model approach in reliability theory is stated as follows: Recruitment is made whenever the cumulative loss of man hours exceeds a breakdown threshold.

Several stochastic models of manpower system have been proposed and studied by many authors [3], [4] and [5] extensively in the past. More specifically [8] have initiated the study on finding the expected time to recruitment for a single grade manpower system using shock model approach in reliability theory. Later [2] have studied the problem of time to recruitment, when the depletion of manpower is classified into exits of personnel and frequent breaks taken by the existing workers and the loss of manpower due to exits forms a sequence of exchangeable and constantly correlated exponential random variables. Recently [1] have studied the problem of time to recruitment when the loss of manpower due to frequent breaks taken by the existing workers forms a modified renewal process. In this paper exits of personnel is classified into voluntary and involuntary exits which forms different renewal process and the inter-breaking decisions forms a modified renewal process. By assuming that the loss of manpower due to voluntary exits forms a sequence of exchangeable and constantly correlated exponential random variables, a stochastic model is constructed and the inter-voluntary exit times are in the following cases: (Case-I) as a sequence of independent and identically distributed exponential random variables, (Case-II) as a sequence of exchangeable and constantly correlated exponential random variables and (Case-III) as a geometric process. Mean and variance of time to recruitment is obtained using an univariate CUM policy of recruitment by assuming specific distribution for the loss of manpower and thresholds. The results are illustrated and specific conclusions are given on the influence of parameters over the performance measures.

2. MODEL DESCRIPTION

Consider an organization with single grade in which exit of personnel takes place either by voluntary and involuntary exit of personnel from the organization or due to breaks taken by the existing workers in the organization. Let X_{I_q}, q = 1,2,3,… be a sequence of exponential random variables denoting the loss of manpower due to the q[th] involuntary exit of personnel from the organization with Laplace transform $\bar{f}_{X_I}(.)$ with parameter $\alpha_1, \alpha_1 > 0$. Let X_{V_r}, r = 1,2,3,… be a sequence of exponential random variables denoting the loss of manpower due to the r[th] voluntary exit of personnel from the organization with Laplace transform $\bar{f}_{X_V}(.)$ with parameter $\alpha_2, \alpha_2 > 0$, correlation ρ' and the relation $v' = \rho'(1-\alpha_2)$. Let Y_l, l = 1,2,3,… be a sequence of exponential random variables denoting the loss of manpower due to the l[th] break taken by the existing personnel from the organization with Laplace transform $\bar{g}_{Y_l}(.)$ with parameter $\gamma, \gamma > 0$. The loss of manpower are assumed to be linear and cumulative. Let $\widetilde{X_{I_q}}$ be the cumulative loss of manpower in the first q involuntary exits of personnel from the organization. Let $\widetilde{X_{V_r}}$ be the cumulative loss of manpower in the first r voluntary exits of personnel from the organization. Let $\widetilde{Y_l}$ be the cumulative loss of manpower in the first l breaks. Let U_{I_q} ; q = 1,2,3… be independent and identically distributed exponential random variables representing the time between q-1[th] and q[th] involuntary exit of personnel from the organization with mean $\frac{1}{\lambda_1}, \lambda_1 > 0$. Let U_{V_r} ; r = 1,2,3,… be independent exponential random variables representing the time between r-1[th] and r[th] voluntary exit of personnel from the organization with mean $\frac{1}{\lambda_2}, \lambda_2 > 0$. Let V_l; l = 1,2,3,… be independent and identically distributed exponential random variables representing the time between l-1[th] and l[th] break with mean $\frac{1}{\beta_1}, \beta_1 > 0$ and forms a modified renewal process with parameter β_2, ($\beta_2 = p\beta_1$), $\beta_2 > 0$ and $0<p<1$ in the sense that, before there is any loss in manpower, every breaking decision has a fixed probability p of causing loss in manpower in the organization. After the first occurrence of loss in manpower, p changes to 1. Let $N_{11}(t)$ be the number of involuntary exit decisions in (0,t], $N_{12}(t)$ be the number of voluntary exit decisions in (0,t] and $N_2(t)$ be the number of breaking decisions in (0,t]. Let Z_{11} be the exponential threshold for the cumulative loss of manpower due to involuntary exits of personnel from the organization with parameter $\theta_{11} > 0$, Z_{12} be the exponential threshold for the cumulative loss of manpower due to voluntary exits of personnel from the organization with parameter $\theta_{12} > 0$ and Z_2 be the exponential threshold for the cumulative loss of manpower due to breaks with parameter $\theta_2 > 0$. Let $Z_{11}+Z_{12}+Z_2$ be the breakdown threshold for the organization. Let W be the time to recruitment for the organization with the distribution function L(.), density function l(.) and the Laplace transform $\bar{l}(.)$. It is assumed that the loss of manhours due to involuntary exits, voluntary exits and breaks, inter-involuntary and voluntary exit times, inter-breaking decision times and breakdown threshold are stochastically independent.

2.1. *Analytical Results*

Let the event {W > t} be the time to recruitment which occurs beyond the time t, and $\{\widetilde{X_{I_{N_{11}(t)}}} + \widetilde{X_{V_{N_{12}(t)}}} + \widetilde{Y_{N_2(t)}} < Z_{11} + Z_{12} + Z_2\}$ represent the event that the cumulative loss of manhours due to the three types of decisions does not cross the breakdown threshold upto the time t. It is proved that the occurrence of these two events are equal.

Hence

$$\{W > t\} \Leftrightarrow \{X_{I_{N_{11}(t)}} + X_{V_{N_{12}(t)}} + Y_{N_2(t)} < Z_{11} + Z_{12} + Z_2\}$$

Conditioning upon $N_{11}(t), N_{12}(t)$ and $N_2(t)$ by using the result of renewal theory [5], the distribution function and density function for time to recruitment are derived. Hence the r[th] moment for the time to recruitment is determined by taking the r[th] derivative with respect to s for the Laplace transform of the time to recruitment at s = 0.

Case-I:

Inter voluntary-exit times are assumed to form a sequence of independent and identically distributed exponential random variables. Taking derivative for the Laplace transform of the random variable W with respect to s and at s = 0, the mean time to recruitment for the present case is derived.

$$E(W) = C_1D_1 - C_2D_2 + C_3D_3 \tag{1}$$

where

$$D_1 = \frac{((1-A_3)h_1 - \mu_1\mu_2)}{m_1\mu_2\mu_1}\sum_{r=0}^{\infty}(\overline{f_{U_V}}(m_1)^{r+1} - \overline{f_{U_V}}(m_1)^r)\overline{f_{X_{V_r}}}(\theta_{11})$$
$$+\frac{(1-A_3)(\mu_1h_2 - h_1)}{m_2(\mu_2-\mu_1)\mu_1}\sum_{r=0}^{\infty}(\overline{f_{U_V}}(m_2)^{r+1} - \overline{f_{U_V}}(m_2)^r)\overline{f_{X_{V_r}}}(\theta_{11})$$
$$+\frac{(1-A_3)(h_1 - \mu_2h_2)}{m_3(\mu_2-\mu_1)\mu_2}\sum_{r=0}^{\infty}(\overline{f_{U_V}}(m_3)^{r+1} - \overline{f_{U_V}}(m_3)^r)\overline{f_{X_{V_r}}}(\theta_{11})$$

$$D_2 = \frac{((1-A_6)h_1 - \mu_1\mu_2)}{m_4\mu_2\mu_1}\sum_{r=0}^{\infty}(\overline{f_{U_V}}(m_4)^{r+1} - \overline{f_{U_V}}(m_4)^r)\overline{f_{X_{V_r}}}(\theta_{12})$$
$$+\frac{(1-A_6)(\mu_1h_2 - h_1)}{m_5(\mu_2-\mu_1)\mu_1}\sum_{r=0}^{\infty}(\overline{f_{U_V}}(m_5)^{r+1} - \overline{f_{U_V}}(m_5)^r)\overline{f_{X_{V_r}}}(\theta_{12})$$
$$+\frac{(1-A_6)(h_1 - \mu_2h_2)}{m_6(\mu_2-\mu_1)\mu_2}\sum_{r=0}^{\infty}(\overline{f_{U_V}}(m_6)^{r+1} - \overline{f_{U_V}}(m_6)^r)\overline{f_{X_{V_r}}}(\theta_{12})$$

$$D_3 = \frac{((1-A_9)h_1 - \mu_1\mu_2)}{m_7\mu_2\mu_1}\sum_{r=0}^{\infty}(\overline{f_{U_V}}(m_7)^{r+1} - \overline{f_{U_V}}(m_7)^r)\overline{f_{X_{V_r}}}(\theta_2)$$
$$+\frac{(1-A_9)(\mu_1h_2 - h_1)}{m_8(\mu_2-\mu_1)\mu_1}\sum_{r=0}^{\infty}(\overline{f_{U_V}}(m_8)^{r+1} - \overline{f_{U_V}}(m_8)^r)\overline{f_{X_{V_r}}}(\theta_2)$$
$$+\frac{(1-A_9)(h_1 - \mu_2h_2)}{m_9(\mu_2-\mu_1)\mu_2}\sum_{r=0}^{\infty}(\overline{f_{U_V}}(m_9)^{r+1} - \overline{f_{U_V}}(m_9)^r)\overline{f_{X_{V_r}}}(\theta_2)$$

The second moment of the random variable W is derived by differentiating twice the Laplace transform of W with respect to s and at s = 0. From these results the variance of time to recruitment for the present case is determined.

Theorem-2.2

Let $Z_i, i = 1,2,3,...k$ be a sequence of exchangeable and constantly correlated exponential random variables with the correlation ρ, $\rho \in [-1,1]$, mean u and the relation v = u(1- ρ). If the probability density function of Zi; i = 1,2,3,…k is $\frac{1}{u}e^{-\left(\frac{t}{u}\right)}, u > 0, 0 < t < \infty$ and the k-fold convolution of the distribution of these random variables is

$$Z_K(t) = \frac{1-\rho}{1-\rho+k\rho}\sum_{i=0}^{\infty}\left(\frac{1-\rho}{1-\rho+k\rho}\right)^i\left(1-\sum_{j=0}^{k+i-1}e^{-\left[\frac{t}{u}\right]}\frac{\int_0^{\frac{t}{u}}\left(e^{-z}z^{k+i-1}\right)dz}{(k+i-1)!}\right)$$

then the Laplace Stieltje's transform for $Z_K(t)$ is

$$\overline{Z_K}(\mathrm{s}) = \frac{(1-\rho)(vs+1)^{1-k}}{(1-\rho)(vs+1)+k\rho vs}$$

Proof:

By taking the Laplace transform and using the relation of Laplace transform and the Laplace Stieltje's transform,

$$\overline{Z_K}(\mathrm{s}) = \frac{s(1-\rho)}{1-\rho+k\rho}\sum_{i=0}^{\infty}\left(\frac{1-\rho}{1-\rho+k\rho}\right)^i\left(\frac{1}{s}-\sum_{j=0}^{k+i-1}\frac{(k+i-j-1)!}{v^{k+i-j-1}\left(s+\left(\frac{1}{v}\right)\right)^{k+i-j}(k+i-j-1)!}\right)$$

Simplifying the above expression, then the Laplace Stieltje's transform for $Z_k(t)$ is reduced to

$$\overline{Z_K}(\mathrm{s}) = \frac{(1-\rho)}{1-\rho+k\rho}\sum_{i=0}^{\infty}\left(\frac{1-\rho}{1-\rho+k\rho}\right)^i(1+vs)^{-(k+i)}$$

Doing further simplifications the Laplace Stieltje's transform is proved. Henceforth the moments are derived by taking the r^th derivative with respect to s for the Laplace Stieltje's transorm of $Z_k(t)$ at s=0. □

***Case-II**:*

In case-II inter voluntary-exit times are assumed to form a sequence of exchangeable and constantly correlated exponential random variables with mean λ_2, correlation ρ with the relation v = λ_2 (1- ρ).

Using the Theorem-2.2 taking derivative with respect to s for the Laplace transform of W at s=0 gives the mean time to recruitment for the present case.

$$E(W) = C_1D_4 - C_2D_5 + C_3D_6 \qquad (2)$$

where

$$D_4 = \frac{((1-A_3)\,h_1 - \mu_1\mu_2)}{m_1\mu_2\mu_1}\sum_{r=0}^{\infty}(\overline{f_{U_{Vr+1}}}(m_1) - \overline{f_{U_{V_r}}}(m_1))\overline{f_{X_{V_r}}}(\theta_{11})$$
$$+\frac{(1-A_3)(\mu_1h_2 - h_1)}{m_2(\mu_2-\mu_1)\mu_1}\sum_{r=0}^{\infty}(\overline{f_{U_{Vr+1}}}(m_2) - \overline{f_{U_{V_r}}}(m_2))\overline{f_{X_{V_r}}}(\theta_{11})$$
$$+\frac{(1-A_3)(h_1 - \mu_2h_2)}{m_3(\mu_2-\mu_1)\mu_2}\sum_{r=0}^{\infty}(\overline{f_{U_{Vr+1}}}(m_3) - \overline{f_{U_{V_r}}}(m_3))\overline{f_{X_{V_r}}}(\theta_{11})$$

$$D_5 = \frac{((1-A_6)\,h_1 - \mu_1\mu_2)}{m_4\mu_2\mu_1}\sum_{r=0}^{\infty}(\overline{f_{U_{Vr+1}}}(m_4) - \overline{f_{U_{V_r}}}(m_4))\overline{f_{X_{V_r}}}(\theta_{12})$$
$$+\frac{(1-A_6)(\mu_1h_2 - h_1)}{m_5(\mu_2-\mu_1)\mu_1}\sum_{r=0}^{\infty}(\overline{f_{U_{Vr+1}}}(m_5) - \overline{f_{U_{V_r}}}(m_5))\overline{f_{X_{V_r}}}(\theta_{12})$$
$$+\frac{(1-A_6)(h_1 - \mu_2h_2)}{m_6(\mu_2-\mu_1)\mu_2}\sum_{r=0}^{\infty}(\overline{f_{U_{Vr+1}}}(m_6) - \overline{f_{U_{V_r}}}(m_6))\overline{f_{X_{V_r}}}(\theta_{12})$$

$$D_6 = \frac{((1-A_9)\,h_1 - \mu_1\mu_2)}{m_7\mu_2\mu_1}\sum_{r=0}^{\infty}(\overline{f_{U_{Vr+1}}}(m_7) - \overline{f_{U_{V_r}}}(m_7))\overline{f_{X_{V_r}}}(\theta_2)$$
$$+\frac{(1-A_9)(\mu_1h_2 - h_1)}{m_8(\mu_2-\mu_1)\mu_1}\sum_{r=0}^{\infty}(\overline{f_{U_{Vr+1}}}(m_8) - \overline{f_{U_{V_r}}}(m_8))\overline{f_{X_{V_r}}}(\theta_2)$$
$$+\frac{(1-A_9)(h_1 - \mu_2h_2)}{m_9(\mu_2-\mu_1)\mu_2}\sum_{r=0}^{\infty}(\overline{f_{U_{Vr+1}}}(m_9) - \overline{f_{U_{V_r}}}(m_9))\overline{f_{X_{V_r}}}(\theta_2)$$

Now differentiating twice the Laplace transform of W with respect to s and at s = 0, the second moment of W is determined. From these results the variance of time to recruitment for the present case is determined.

Theorem-2.3

Let Z_k be a sequence of non-negative random variables and 'a' be a positive constant. If a normalized stochastic process $\{V_k\}_{k=1}^{\infty}$, where $V_k = a^{k-1}Z_k$, k=1,2,3,… is a geometric process with a parameter 'a', then the Laplace transform for V_k is

$$\overline{V_K}(\mathrm{s}) = \prod_{r=1}^{k}\overline{f}\left(\frac{s}{a^{r-1}}\right)$$

Proof:

The distribution function $V_k(t) = F(a^{k-1}t)$; k = 1,2,3,… is determined from the definition of geometric process and by differentiating the distribution function with respect to t, the density function of $v_k(t) = a^{k-1}f(a^{k-1}t)$; k = 1, 2, 3, … is derived.

Now using the property, *Laplace transform of the convolution of random variables is the product of their Laplace transforms*, the Laplace transform for $v_k(t)$ is derived. Hence the theorem is proved. □

Corollary: If a = 1, then the sequence of random variables V_k, k = 1, 2, 3; … forms an ordinary renewal process.

Remark: If a > 1, $\{V_k\}_{k=1}^{\infty}$ is stochastically decreasing and when 0 < a < 1, $\{V_k\}_{k=1}^{\infty}$ forms a stochastically increasing sequence.

Case-III:

In this case inter voluntary-exit times are assumed to form a geometric process with a parameter $a > 0$.

Using Theorem 2.3, the mean time to recruitment for Case-III is determined by taking first derivative with respect to s for the Laplace transform of W at s=0.

$$E(W) = C_1D_7 - C_2D_8 + C_3D_9 \tag{3}$$

$$D_7 = \frac{((1-A_3)\,h_1 - \mu_1\mu_2)}{m_1\mu_2\mu_1}\sum_{r=0}^{\infty}\left(\overline{f_{U_{V_{r+1}}}}(m_1) - \overline{f_{U_{V_r}}}(m_1)\right)\overline{f_{X_{V_r}}}(\theta_{11})$$
$$+\frac{(1-A_3)(\mu_1 h_2 - h_1)}{m_2(\mu_2-\mu_1)\mu_1}\sum_{r=0}^{\infty}\left(\overline{f_{U_{V_{r+1}}}}(m_2) - \overline{f_{U_{V_r}}}(m_2)\right)\overline{f_{X_{V_r}}}(\theta_{11})$$
$$+\frac{(1-A_3)(h_1 - \mu_2 h_2)}{m_3(\mu_2-\mu_1)\mu_2}\sum_{r=0}^{\infty}\left(\overline{f_{U_{V_{r+1}}}}(m_3) - \overline{f_{U_{V_r}}}(m_3)\right)\overline{f_{X_{V_r}}}(\theta_{11})$$

$$D_8 = \frac{((1-A_6)\,h_1 - \mu_1\mu_2)}{m_4\mu_2\mu_1}\sum_{r=0}^{\infty}\left(\overline{f_{U_{V_{r+1}}}}(m_4) - \overline{f_{U_{V_r}}}(m_4)\right)\overline{f_{X_{V_r}}}(\theta_{12})$$
$$+\frac{(1-A_6)(\mu_1 h_2 - h_1)}{m_5(\mu_2-\mu_1)\mu_1}\sum_{r=0}^{\infty}\left(\overline{f_{U_{V_{r+1}}}}(m_5) - \overline{f_{U_{V_r}}}(m_5)\right)\overline{f_{X_{V_r}}}(\theta_{12})$$
$$+\frac{(1-A_6)(h_1 - \mu_2 h_2)}{m_6(\mu_2-\mu_1)\mu_2}\sum_{r=0}^{\infty}\left(\overline{f_{U_{V_{r+1}}}}(m_6) - \overline{f_{U_{V_r}}}(m_6)\right)\overline{f_{X_{V_r}}}(\theta_{12})$$

$$D_9 = \frac{((1-A_9)\,h_1 - \mu_1\mu_2)}{m_7\mu_2\mu_1}\sum_{r=0}^{\infty}\left(\overline{f_{U_{V_{r+1}}}}(m_7) - \overline{f_{U_{V_r}}}(m_7)\right)\overline{f_{X_{V_r}}}(\theta_2)$$
$$+\frac{(1-A_9)(\mu_1 h_2 - h_1)}{m_8(\mu_2-\mu_1)\mu_1}\sum_{r=0}^{\infty}\left(\overline{f_{U_{V_{r+1}}}}(m_8) - \overline{f_{U_{V_r}}}(m_8)\right)\overline{f_{X_{V_r}}}(\theta_2)$$
$$+\frac{(1-A_9)(h_1 - \mu_2 h_2)}{m_9(\mu_2-\mu_1)\mu_2}\sum_{r=0}^{\infty}\left(\overline{f_{U_{V_{r+1}}}}(m_9) - \overline{f_{U_{V_r}}}(m_9)\right)\overline{f_{X_{V_r}}}(\theta_2)$$

Taking second derivative for the Laplace transform of W with respect to s and at s = 0, the second moment for W is derived. From these two results the variance of time to recruitment is determined for the present case.

The expressions for the notations used in the equations (1), (2) and (3) are given below.

$$A_1 = \frac{\alpha_1}{\alpha_1+\theta_{11}}, A_2 = \frac{\alpha_2}{\alpha_2+\theta_{11}}, A_3 = \frac{\gamma}{\gamma+\theta_{11}}, A_4 = \frac{\alpha_1}{\alpha_1+\theta_{12}},$$
$$A_5 = \frac{\alpha_2}{\alpha_2+\theta_{12}}, A_6 = \frac{\gamma}{\gamma+\theta_{12}}, A_7 = \frac{\alpha_1}{\alpha_1+\theta_2}, A_8 = \frac{\alpha_2}{\alpha_2+\theta_2},$$
$$A_9 = \frac{\gamma}{\gamma+\theta_2}; h_1 = \beta_1\beta_2; h_2 = \beta_2; h_3 = \beta_1+\beta_2-\beta_1 A_3;$$
$$h_4 = \beta_1\beta_2 - \beta_1\beta_2 A_3;$$

$$m_1 = \lambda_1(1-A_1)+\lambda_2(1-A_2); m_2 = \lambda_1(1-A_1)+\lambda_2(1-A_2)+\mu_1;$$
$$m_3 = \lambda_1(1-A_1)+\lambda_2(1-A_2)+\mu_2; m_4 = \lambda_1(1-A_4)+\lambda_2(1-A_5);$$
$$m_5 = \lambda_1(1-A_4)+\lambda_2(1-A_5)+\mu_1; m_6 = \lambda_1(1-A_4)+\lambda_2(1-A_5)+\mu_2;$$
$$m_7 = \lambda_1(1-A_7)+\lambda_2(1-A_8); m_8 = \lambda_1(1-A_7)+\lambda_2(1-A_8)+\mu_1;$$
$$m_9 = \lambda_1(1-A_7)+\lambda_2(1-A_8)+\mu_2;$$
$$C_1 = \frac{\theta_{12}\theta_2}{(\theta_{12}-\theta_{11})(\theta_2-\theta_{11})}; C_2 = \frac{\theta_{11}\theta_2}{(\theta_{12}-\theta_{11})(\theta_2-\theta_{12})};$$
$$C_3 = \frac{\theta_{12}\theta_{11}}{(\theta_2-\theta_{11})(\theta_2-\theta_{12})};$$

and

$$\theta_{11} \neq \theta_{12} \neq \theta_2, \mu_1 \neq \mu_2, \beta_1 \neq \beta_2$$
$$A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9 \neq 1, .$$

2.4 NUMERICAL ILLUSTRATION:

The influence of nodal parameters on the performance measures namely mean and variance of the time to recruitment is studied numerically using MATLAB. The performance measures are calculated for all the three cases.

Case-I:

Effect of λ_1, λ_2, α_1, α_2, γ , ρ' for the mean and variance of time to recruitment is studied by fixing the value of the parameters $\beta_1 = 0.2$; p = 0.4; $\theta_{11} = 0.7$; $\theta_{12} = 0.8$, $\theta_2 = 0.9$, r=5.

λ_1	λ_2	α_1	α_2	γ	ρ'	E(W)	V(W)
0.5	0.6	0.1	0.1	0.2	0.3	0.0119`	1.8904
0.5	0.6	0.1	0.1	0.2	-0.5	0.0120	1.8895
0.5	0.6	0.1	0.1	0.2	-0.3	0.0119	1.8904
0.5	0.6	0.1	0.1	0.3	0.3	0.0123	1.9070
0.5	0.6	0.1	0.1	0.4	0.3	0.0127	1.9203
0.5	0.6	0.1	0.1	0.5	0.3	0.0130	1.9313
0.5	0.6	0.1	0.2	0.2	0.3	0.0282	1.3974
0.5	0.6	0.1	0.3	0.2	0.3	0.0498	0.9976
0.5	0.6	0.1	0.4	0.2	0.3	0.0745	0.6652
0.5	0.6	0.2	0.1	0.2	0.3	0.0167	1.4919
0.5	0.6	0.3	0.1	0.2	0.3	0.0207	1.1880
0.5	0.6	0.4	0.1	0.2	0.3	0.0243	0.9516
0.5	0.7	0.1	0.1	0.2	0.3	0.0127	2.4349
0.5	0.8	0.1	0.1	0.2	0.3	0.0135	2.9123
0.5	0.9	0.1	0.1	0.2	0.3	0.0141	3.3450
0.6	0.6	0.1	0.1	0.2	0.3	0.0092	2.1925
0.7	0.6	0.1	0.1	0.2	0.3	0.0072	2.4619
0.8	0.6	0.1	0.1	0.2	0.3	0.0057	2.7036

Case-II:

Effect of λ_1, λ_2, α_1, α_2, γ, ρ, ρ' for the mean and variance of time to recruitment is studied by fixing the value of the parameters $\beta_1 = 0.2$; p = 0.01; $\theta_{11} = 0.7$;
$\theta_{12} = 0.8$, $\theta_2 = 0.9$.

λ_1	λ_2	α_1	α_2	γ	ρ'	ρ	E(W)	V(W)
0.5	0.4	0.4	0.1	0.2	0.3	-0.3	0.5866	5.6697
0.5	0.4	0.4	0.1	0.2	0.3	-0.2	0.5000	5.6818
0.5	0.4	0.4	0.1	0.2	0.3	0.2	0.4903	5.1414
0.5	0.4	0.4	0.1	0.2	0.3	0.3	0.5532	4.9107
0.5	0.4	0.4	0.1	0.2	-0.3	0.2	0.4901	5.1415
0.5	0.4	0.4	0.1	0.2	-0.4	0.2	0.4874	5.1435
0.5	0.4	0.4	0.1	0.2	0.5	0.2	0.4848	5.1454
0.5	0.4	0.4	0.1	0.2	0.4	0.2	0.4879	5.1432
0.5	0.4	0.4	0.1	0.22	0.3	0.2	0.3424	5.2380
0.5	0.4	0.4	0.1	0.24	0.3	0.2	0.1941	5.2928
0.5	0.4	0.4	0.1	0.26	0.3	0.2	0.0455	5.3053
0.5	0.4	0.4	0.2	0.2	0.3	0.2	1.4113	4.1948
0.5	0.4	0.4	0.3	0.2	0.3	0.2	2.0854	2.4197
0.5	0.4	0.4	0.4	0.2	0.3	0.2	2.5852	0.5042
0.5	0.4	0.5	0.1	0.2	0.3	0.2	1.0744	4.6826
0.5	0.4	0.6	0.1	0.2	0.3	0.2	1.6251	3.5868
0.5	0.4	0.7	0.1	0.2	0.3	0.2	2.1417	1.9767
0.5	0.35	0.4	0.1	0.2	0.3	0.2	1.7818	2.1859
0.5	0.37	0.4	0.1	0.2	0.3	0.2	1.2364	3.8586
0.5	0.39	0.4	0.1	0.2	0.3	0.2	0.7285	4.8586
0.51	0.4	0.4	0.1	0.2	0.3	0.2	0.3580	5.1981
0.52	0.4	0.4	0.1	0.2	0.3	0.2	0.2294	5.2186
0.53	0.4	0.4	0.1	0.2	0.3	0.2	0.1043	5.2087

Case-III:

Effect of λ_1, λ_2, α_1, α_2, γ, ρ', a for the mean and variance of time to recruitment is studied by fixing the value of the parameters $\beta_1 = 0.3$; p = 0.4; $\gamma = 0.2$;
$\theta_{11} = 0.001$; $\theta_{12} = 0.002$, $\theta_2 = 0.003$

λ_1	λ_2	α_1	α_2	γ	ρ'	a	E(W)	V(W)
0.8	0.5	0.4	0.1	0.1	0.4	0.6	0.7478	2.9278
0.8	0.5	0.4	0.1	0.1	0.4	0.8	0.7571	2.7633
0.8	0.5	0.4	0.1	0.1	0.4	2	0.8217	1.8055
0.8	0.5	0.4	0.1	0.1	0.4	3	0.8525	1.4204
0.8	0.5	0.4	0.1	0.1	0.5	0.6	0.7478	2.9278
0.8	0.5	0.4	0.1	0.1	0.6	0.6	0.7478	2.9278
0.8	0.5	0.4	0.1	0.1	-0.1	0.6	0.7478	2.9278
0.8	0.5	0.4	0.1	0.1	-0.2	0.6	0.7478	2.9278
0.8	0.5	0.4	0.1	0.2	0.4	0.6	0.4938	3.1329
0.8	0.5	0.4	0.1	0.25	0.4	0.6	0.3653	3.2016
0.8	0.5	0.4	0.1	0.3	0.4	0.6	0.2360	3.2440
0.8	0.5	0.4	0.2	0.1	0.4	0.6	0.5167	3.8257
0.8	0.5	0.4	0.3	0.1	0.4	0.6	0.3274	4.4616
0.8	0.5	0.4	0.4	0.1	0.4	0.6	0.1708	4.9101
0.8	0.5	0.5	0.1	0.1	0.4	0.6	0.5440	3.6143
0.8	0.5	0.6	0.1	0.1	0.4	0.6	0.3632	4.1604
0.8	0.5	0.7	0.1	0.1	0.4	0.6	0.1995	4.5946
0.8	0.6	0.4	0.1	0.1	0.4	0.6	1.1136	1.6311
0.8	0.65	0.4	0.1	0.1	0.4	0.6	1.2659	1.0166
0.8	0.7	0.4	0.1	0.1	0.4	0.6	1.4016	0.4329
0.75	0.5	0.4	0.1	0.1	0.4	0.6	0.5961	3.3124
0.65	0.5	0.4	0.1	0.1	0.4	0.6	0.2725	3.9921
0.6	0.5	0.4	0.1	0.1	0.4	0.6	0.1006	4.2755

CONCLUSIONS:

In Case-(I), increasing γ (λ_2) by fixing the other parameters the mean and variance of time to recruitment increases, increasing α_1 (α_2) by fixing the other parameters the mean time to recruitment increases and variance of time to recruitment decreases. When increasing λ_2 (ρ') by fixing the other parameters the mean time to recruitment decreases and variance of time to recruitment increases.

In Case-(II), increasing α_2 (α_1) by fixing the other parameters the mean time to recruitment increases and variance of time to recruitment decreases. When increasing γ (λ_1, λ_2) by fixing the other parameters the mean time to recruitment decreases and variance of time to recruitment increases. While increasing the positive value of ρ' (negative value of ρ') the mean time to recruitment increases and the variance of time to recruitment decreases. Increasing the positive value of ρ' (negative value of ρ) the mean time to recruitment decreases and the variance of time to recruitment increases

In Case-(III), for (a > 1, a < 1) increasing the geometric process parameter a (λ_2) by fixing the other parameters the mean time to recruitment increases and variance of time to recruitment decreases. When increasing γ (α_2, α_1) and decreasing λ_1 by fixing the other parameters the mean time to recruitment decreases and variance of time to recruitment increases. If the positive and negative value of ρ increases the mean and variance of time to recruitment remains constant.

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